Meta-Analysis

- What is it? Why is it important?
- How do you do it? (Summer)

What is meta-analysis?

- Meta-analysis can be thought of as a form of survey research in which research reports are the units surveyed (Lipsey and Wilson, 2001, Practical Meta-Analysis, Sage)
- Meta-analysis is the quantitative integration of research that is a special form of systematic research synthesis
- Meta-analysis can be thought of as an approach to the quantitative analysis of replications

Good books on meta-analysis

- Lipsey and Wilson, (2001), Practical Meta-Analysis, Sage. (Easy to read, very practical)
- Glass, McGaw, and Smith, (1981), Meta-Analysis in Social Research, Sage. (A classic)
- Cooper and Hedges, (1994), Handbook of Research Synthesis, Russell Sage Foundation. (Very comprehensive, technical, a must for any meta-analyst)

What types of research questions can be addressed in a meta-analysis?

Types of research questions addressed in meta-analysis

- What does the research in a particular area tell us about….?
- Does cognitive-behavior therapy decrease depression? (Gaffan, Tsaousis, and Kemp-Wheeler, "Researcher allegiance and meta-analysis: The case of cognitive therapy for depression," (1995), Journal of Consulting and Clinical Psychology, 63(6), 966-980).
- Is there a relationship between being sexually abused as a child and later psychopathology? (Rind, Tromovich, and Bauserman, "A meta-analytic examination of assumed properties of child sexual abuse using college samples", (1998), Psychological Bulletin, 124(10), 22-53).

- Is there a relationship between participation in victim-offender mediation and subsequent delinquent behavior? (Nugent, Williams-Hayes, and Umbreit, in press, Research on Social Work Practice).
- What study characteristics moderate effect size magnitude?
- Substantive questions about some phenomena
- Questions about which methodological characteristics contribute the variability in outcomes

Why is Understanding Meta-Analysis Important

The use of systematic research reviews as a tool for identifying "best practices" is becoming more and more prominent. Meta-analysis is rapidly becoming a principal method for conducting systematic reviews.

How is Meta-Analysis Done?

Steps in a meta-analysis

- Research question/problem formulation
- Retrieval of research studies
- Effect size selection
- Identification and coding of independent variables
- Data analysis
- Interpreting and understanding results
- Writing up results

Create a Literature Search Record

- Include sources searched
- Include citations found; citations retrieved and how; citations not retrieved and methods used to get them
- Include personal contacts with other researchers and results
- Include advertisements used
- Include how world wide web searched done and results

Five Literature Search methods

- Footnote chasing
  - References in nonreview papers in journals
  - References in review papers
  - References in books
  - Topical bibliographies
- Consultation
  - Informal conversations
  - Communication with fellow researchers
  - Formal requests from other researchers
  - General requests to government agencies

- Searches in subject indexes
  - Manual search of abstract data bases
  - Computer search of abstract data bases (eg., PsychInfo, ERIC, etc.)
- Browsing
  - Browsing through libraries
- Citation searches
  - Manual search of citation index
  - Computer search of citation index (eg., SSCI)

Variables involved in a meta-analysis

- Dependent – one or more measures of "effect size"

- Independent Variables
  - study characteristics
    - methodological quality;
    - sampling methods;
    - group formation methodology;
    - measurement;
    - etc.
  - subject characteristics
    - age;
    - gender;
    - ethnicity;
    - etc.

  - treatment variables
    - treatment type;
    - type of comparison group (eg., placebo; no-treatment; etc.)
  - context variables
    - location of study;
    - type of supervision of therapist;
    - etc.
  - researcher characteristics
    - therapeutic allegiance;
    - experience;
    - education level;
    - etc.

Effect sizes

- An effect size is a statistic which embodies information about either the direction or magnitude (or both) of quantitative research findings (Lipsey & Wilson, 2001)
- Effect sizes used in a meta-analysis are considered to be “metric free”
- Just about any statistic can, in principal, be considered as an “effect size”

Effect size statistics

- Single variable
- Two variable
  - D-family
  - R-family
  - Odds-ratio

Single variable effect sizes – Statistics that describe

Angel

Single variable – the mean

$$ES_m = \bar{X}$$

$$SE_m = \frac{s}{\sqrt{n}}$$

$$w_m = \frac{1}{SE_m^2} = \frac{n}{s^2}$$

Example

$\bar{X} = 3.22$

$s = 3.57$

$n = 78$

$$SE_m = \frac{3.57}{\sqrt{78}} = .404$$

$$w_m = \frac{1}{.404^2} = 6.13$$

Single variable - The logit

$$ES_l = \ln\left[\frac{p}{1-p}\right]$$

$$SE_l = \sqrt{\frac{1}{np} + \frac{1}{n(1-p)}}$$

$$w_l = \frac{1}{SE_l^2} = np(1-p)$$

Single variable – the standard deviation

$$ES_{sd} = \ln(s) + [\frac{1}{2(n-1)}]$$

$$SE_{sd} = \sqrt{\frac{1}{2(n-1)}}$$

$$w_{sd} = 2(2n-1)$$

The d-family

Marmaduke

Standardized mean difference

$$ES_{sm} = \frac{\overline{X}_{G2} - \overline{X}_{G1}}{SD}$$

IF N < 20

$$ES'_{SM} = \left[1 - \frac{3}{4N-9}\right] ES_{SM}$$

$$SE_{sm} = \sqrt{\frac{n_{G1} + n_{G2}}{n_{G1} n_{G2}} + \frac{(ES'_{sm})^2}{2(n_{G1} + n_{G2})}}$$

$$w_{sm} = \frac{1}{SE_{sm}^2}$$

Computing ES_{sm} from statistical tests of significance

$$ES_{sm} = t\sqrt{\frac{n_{G1} + n_{G2}}{n_{G1} n_{G2}}}$$

$$\left|ES_{sm}\right| = \sqrt{\frac{F(n_{G1} + n_{G2})}{n_{G1} n_{G2}}}$$

ES_{sm} from a phi coefficient

$$ES_{sm} = \frac{2r}{\sqrt{1 - r^2}}$$

BY CONVENTION, WHEN TREATMENT AND CONTROL GROUPS ARE CONTRASTED, A + SIGN IS GIVEN TO AN EFFECT SIZE TO INDICATE THE TREATMENT GROUP DID BETTER THAN THE COMPARISON GROUP

The r-family of effect sizes: Indices of correlational association

Paisley

$$ES_r = r$$

$$ES_{Z_r} = \frac{1}{2}\ln\left[\frac{1+r}{1-r}\right]$$

$$SE_{Z_r} = \frac{1}{\sqrt{n-3}}$$

$$w_{Z_r} = \frac{1}{SE^2_{Z_r}} = n-3$$

Computing ES_r from t-test results

$$ES_r = \frac{t}{\sqrt{t^2 + df}} = \frac{t}{\sqrt{t^2 + (n - 2)}}$$

$$ES_r = \frac{t}{\sqrt{t^2 + n_1 + n_2 - 2}}$$

Point-biserial correlation effect size from ES_{sm}

$$ES_{r_{pb}} = \frac{ES_{sm}}{\sqrt{4 + ES_{sm}^2}}$$

Effect size statistics for dichotomous outcomes

The odds-ratio

- The odds-ratio is a statistic that compares two groups in terms of the relative odds of an event or outcome

$$odds = \frac{p}{1 - p}$$

	no recidivism	recidivated
Control group	A	B
Treatment group	C	D

$$ES_{OR} = \frac{AD}{BC} = \frac{p_a(1 - p_c)}{p_c(1 - p_a)}$$

Natural log odds-ratio

$$ES_{LOR} = \ln\left(ES_{OR}\right)$$

$$SE_{LOR} = \sqrt{\frac{1}{n_a} + \frac{1}{n_b} + \frac{1}{n_c} + \frac{1}{n_d}}$$

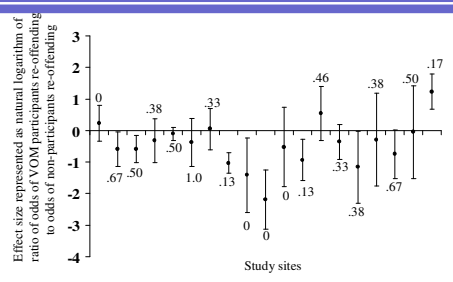
$$w_{LOR} = \frac{1}{SE_{LOR}^2}$$

$$ES_{LOR} = \ln\left[\frac{p_{G1}}{1 - p_{G1}}\right] - \ln\left[\frac{p_{G2}}{1 - p_{G2}}\right]$$

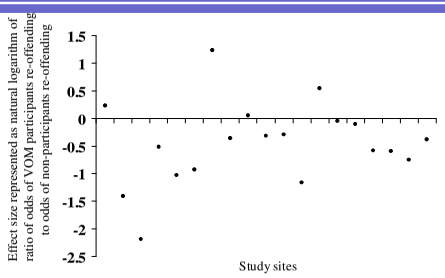
Data analysis methods

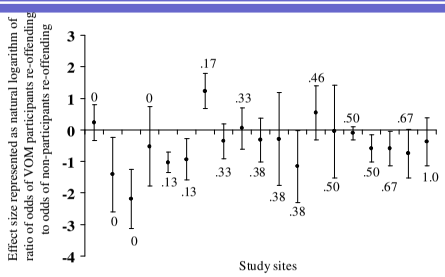
Graphical methods

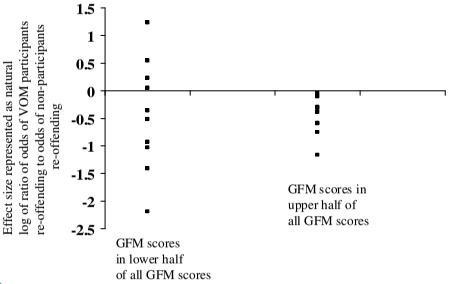
Plot of effect sizes with 95% confidence intervals

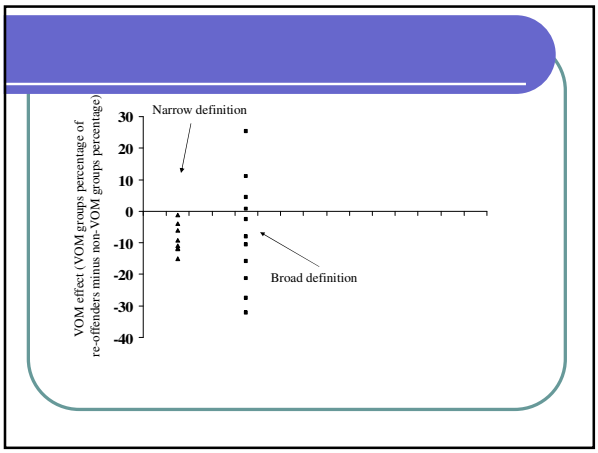


Plot of effect sizes versus group formation methodological quality









The use of weighted least squares regression

Statistical analysis methods

- Fixed effects models: have fixed parameters plus a single residual term
- Random effects models: have two residual terms
- Mixed models: have fixed parameters plus two residual terms

Data analysis – steps in analyzing a distribution of effect sizes

- Create set of independent effect sizes
- Compute weighted mean, weighting by inverse variance weights
- Determine confidence interval for mean
- Test for homogeneity of distribution
- If heterogeneous distribution, conduct further analyses
- Weighted least squares regression (fixed effects)
- HLM (random effects; mixed models)

The mean effect size and 95% confidence interval

$$\overline{ES} = \frac{\sum (w_i ES_i)}{\sum w_i}$$

$$SE_{\overline{ES}} = \sqrt{\frac{1}{\sum w_i}}$$

$$\overline{ES_L} = \overline{ES} - z_{(1-\alpha)}(SE_{\overline{ES}})$$

$$\overline{ES_U} = \overline{ES} + z_{(1-\alpha)}(SE_{\overline{ES}})$$

Z = 1.96 for alpha = .05
Z = 2.58 for alpha = .01

Z – test of mean effect size

$$z = \frac{|\overline{ES}|}{SE_{\overline{ES}}} = \frac{\frac{\sum w_i ES_i}{\sum w_i}}{\sqrt{\frac{1}{\sum w_i}}}$$

Compute the following:

1. ES_i
2. SE_{ES}^2
3. $w_i = \frac{1}{SE_{ES}^2}$
4. $w_i \times ES_i = wes_i$
5. $w_i \times ES_i^2 = wessq_i$
6. $w_i^2 = wsq_i$

7. $\overline{ES} = \frac{\sum wes_i}{\sum w_i}$
8. $SE_{\overline{ES}} = \sqrt{\frac{1}{\sum w_i}}$
9. $z = \frac{\overline{ES}}{SE_{\overline{ES}}}$
10. $\overline{ES}_L = \overline{ES} - 1.96(SE_{\overline{ES}})$
    $\overline{ES}_U = \overline{ES} + 1.96(SE_{\overline{ES}})$
11. $Q = \sum wessq_i - \frac{\left(\sum wes_i\right)^2}{\sum w_i}$

    (Q has k-1 df, where k = number of Studies)

A statistically non-significant Q is consistent with homogenous effect sizes; variability in effect sizes is likely due to sampling variability associated with sampling of different subjects in studies

A statistically significant Q is interpreted to mean that variability in effect sizes is greater than would be expected from sampling variability associated with different persons in studies. Three possibilities exist: (1) there is systematic variability in effect sizes in addition to sampling error associated with different subjects; (2) there is an additional random component associated with random variations in studies that cannot be modeled; and (3) a combination of (1) and (2).

If researcher chooses to model a random effects component, then an additional variance component must be added to the squared standard error of the effect size statistic:

$$v_\theta = \frac{Q - (k-1)}{\sum w_i - \left(\frac{\sum wsq_i}{\sum w_i}\right)}$$

So, the new v_i is,

$$v_i^* = v_i + v_\theta$$

and

$$w_i^* = \frac{1}{v_i + v_\theta}$$

Then the terms

$$wes_i$$

$$wessq_i$$

$$wsq_i$$

are recomputed as are the associated sums of these terms; then a new 95% confidence interval for the mean effect size is computed.

Weighted regression analysis

Forrest Gump

1. Conduct weighted least squares regression, using the inverse variance as the regression weight.
2. Conduct homogeneity tests of the regression model and residual variance by:

a. Test of homogeneity of effect sizes

$Q_{Overall}$ = total sums-of-squares with total df as the chi-square df

b. Test of regression model

Q_R = regression sums-of-squares, with regression model df as the chi-square df.

c. Test of residual variation homogeneity

Q_E = residual sums-of-squares, with Residual df as the chi-square df.

d. Test statistical significance of partial regression coefficients by:

$$z = \frac{B}{SE'_B} \quad \text{where} \quad SE'_B = \frac{SE_B}{\sqrt{MSE}}$$

and MSE = mean square residual for the regression model

A Fixed Effects Analysis

ANOVA[b,c]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	71.596	3	23.865	16.468	.000[a]
	Residual	21.738	15	1.449		
	Total	93.334	18			

a. Predictors: (Constant), SCORE1, DELTA, DEF
b. Dependent Variable: EFS
c. Weighted Least Squares Regression - Weighted by WGHT

Coefficients[a,b]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-.976	.269		-3.633	.002
	DELTA	-.038	.006	-.795	-6.349	.000
	DEF	.515	.198	.373	2.599	.020
	SCORE1	.736	.381	.278	1.931	.073

a. Dependent Variable: EFS
b. Weighted Least Squares Regression - Weighted by WGHT

Test of homogeneity overall:

$\chi^2(18) = SSTOTAL = 93.334$

$p < .05$

$\chi^2(3) = MSREGRESSION = 71.596$

$p < .05$

$\chi^2(15) = SSRESIDUAL = 21.738$

$p < .05$

Tests of regression coefficients

1. Coefficient for "delta" $SE_{delta} = \frac{SE_{B_{delta}}}{\sqrt{MSE}} = \frac{.006}{\sqrt{1.449}} = .0049$

$$z = \frac{B_{delta}}{SE_{delta}} = \frac{-.038}{.0049} = -7.76$$

2. Coefficient for "def" (definition)

$$SE_{def} = \frac{SE_{B_{def}}}{\sqrt{MSE}} = \frac{.198}{\sqrt{1.449}} = .1645$$

$$z = \frac{.515}{.1645} = 3.13$$

3. Coefficient for "score1"

$$SE_{score1} = \frac{.381}{\sqrt{1.449}} = .3165$$

$$z = \frac{.736}{.3165} = 2.325$$

A Random Effects Analysis

ANOVA[b,c]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	15.168	4	3.792	9.686	.001[a]
	Residual	5.481	14	.391		
	Total	20.649	18			

a. Predictors: (Constant), nonvom, def, delta, score1

b. Dependent Variable: efs

c. Chi-sq(4) = 15.17, p < .01, Chi-sq(14) = 5.48, p > .05

Coefficients[a,b]

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	Z	Sig.
1	(Constant)	-.679	.423		-.117	>.05
	delta	-.033	.010	-.561	-2.064	<.05
	def	.746	.297	.504	1.571	.116
	score1	1.753	.602	.641	1.821	.069
	nonvom	-.026	.012	-.371	-1.368	>.15

a. Dependent Variable: efs

b. Weighted Least Squares Regression - Weighted by newwght

Vote Counting Methods – Nonparametric approaches

An application of the sign test

1. Set H_0: p = .50; H_1: p > .50
2. Count number of outcomes in "desired" direction
3. Use binomial probability distribution to obtain p-value for obtained count

Example: 15 of 19 outcomes in specified direction, so $\hat{p} = .84$

And associated p-value from binomial table:

.0018+ .0003+ .0000+ .0000 = .0021

Test of combined statistical significance – another nonparametric approach

1. tippet's minimum p : (a) arrange exact p-values from lowest to highest; (b) set critical alpha by:

$\alpha = 1 - (1 - \alpha_*)^{(1/k)}$ (k = n of studies or effect sizes)

where α_* = desired overall type I error rate; (c) compare minimum obtained exact p-value against alpha; (d) if minimum obtained exact alpha < set alpha, then reject null hypothesis that all obtained effect sizes are zero.

Example: Obtained p-values (k = 19) range from .0001 to .9452

$\alpha = 1 - (1 - .05)^{(1/19)} = .00269$

minimum p = .0001 < .00269; reject null hypothesis